Analysis of Complex Modulated Carriers Using Statistical Methods

Richard H. Blackwell, Director of Engineering, Boonton Electronics

Abstract ... This paper describes a method for obtaining and using probability functions to analyze the peak power of complex modulated RF signals.

Peak Power Measurement

The average power of a modulated carrier which has varying amplitude can be measured accurately by a CW type power meter with a thermoelectric detector, or a diode detector if used at low power in its square-law response region. Pulse power was determined traditionally by adjusting the average power reading for the duty cycle of the modulating pulse. This method becomes inaccurate if the pulse shape is not ideal and useless for complex modulation. Recent advances in digital techniques have made it possible to measure peak power as well as average power accurately with total dynamic range and modulation bandwidth as the only limiting conditions. Knowledge of the modulation method or modulating signal is not required. In simplified form a digital peak power measuring system consists of the following: *See Fig. 1*

- A diode detector with wide rf bandwidth and a narrower video bandwidth.
- A log amplifier compatible with the video bandwidth.
- A fast sample and hold asynchronous with respect to the modulation.
- An analog to digital converter which operates at the sampling rate.
- A Digital Signal Processor (DSP) with software program.
- A precision digitally controlled cw power calibrator.

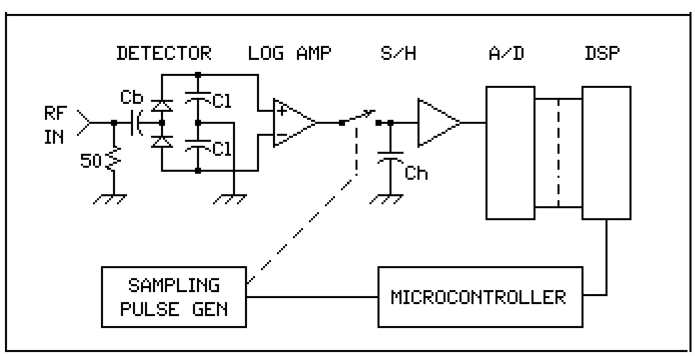

Figure 1. Simplified view of peak power measurement system.

Precision Digitally Controlled Calibrator. The calibration process is a very important part of the measurement system. In order to eliminate the error associated with diode non-linearity, a calibration table is created which stores the output of each detector at a number of precision power levels covering the effective dynamic range. This is accomplished automatically by a precision, digitally controlled rf power source and control program. The resulting calibration table is extended by interpolation to create a power entry for all possible a/d converter values. This in turn yields the power for each individual sample of the detected rf signal. It is this characteristic which separates this method of power measurement from the conventional average power method in which the output of the detector is averaged before conversion.

The random power samples can be processed to provide peak power and average power. It does not matter that the samples are disordered in time. The sum of the random samples over an interval is the same, provided there is no periodic relationship between the sampling rate and the modulating signal. In addition, there must be a sufficient quantity of samples taken to ensure adequate coverage. The advantage of a high sampling rate is the ease of accumulating a large number of sample points for each reading.

If the detected signal is stationary or quasi-stationary in time, the waveform can be re-constructed from the random samples. In conventional pulse or linear amplitude modulation, the rf carrier envelope and thus the detected signal correspond closely to the modulating signal waveform. This correspondence leads naturally to power measurements which relate in the time domain to the demodulated signal.

Statistical Methods

Digital modulation methods in which amplitude and phase modulation are combined in a multi-level arrangement (QUAM) to represent a group of bit values from one or more data streams, multiple carrier systems and spread spectrum techniques do not have simple waveforms which can be directly related to modulation parameters. Parameters such as modulation depth and modulation index are not useful because the peak to average power ratio of the modulated carrier is a complex function of the data stream content, rather than the amplitude of the modulating signal. This situation suggests a statistical approach to analyzing complex modulation.

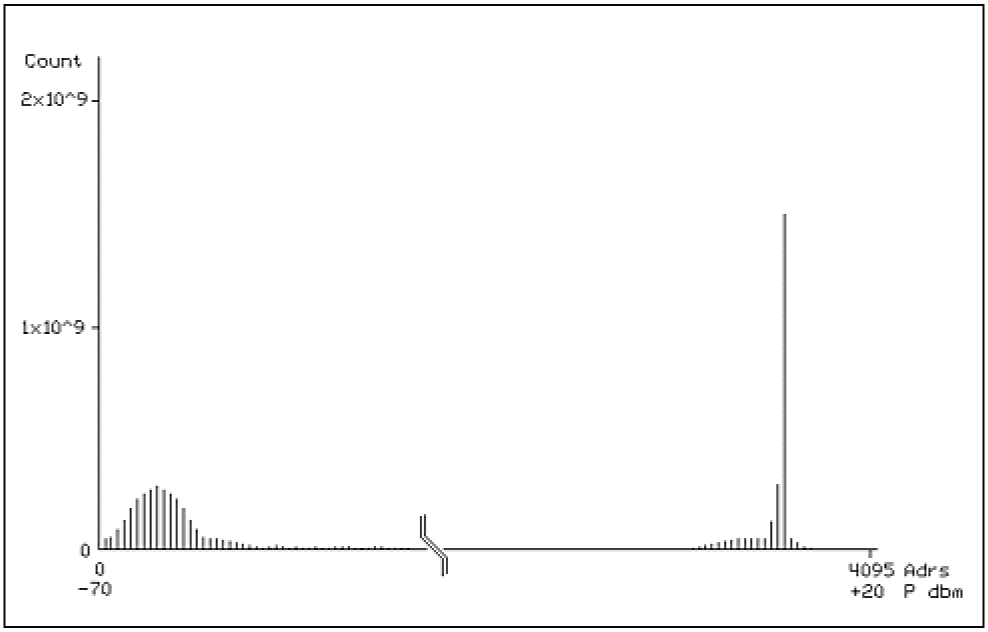

Figure 2. Sample Count Array in memory.

Since the power of the individual random samples is known, they can be sorted and counted by power level. For a 12-bit a/d converter system there are at most 4096 possible power levels. If a memory array of this size is set up, each address corresponds to one of the possible power levels. The value of each sample is taken as an address into the array and the count stored at the location is incremented by one each time a sample is taken. *See Fig. 2*

The measurement process begins by clearing the count array to all zeroes and starting the random sampler. It is necessary to keep track of the total number of samples taken in order to scale the results properly and to estimate the statistical accuracy, which is inversely proportional to the square root of the number of samples. In general, a very large number of samples and a long running time are both desirable. A word size of 31-bits will permit a sample size of at least 2.1 billion (2.1×10^9). Even at 500,000 samples per second, the running time will be 4,200 seconds or 1.17 hours. The measurement could be allowed to run indefinitely with a suitable decimation process. Unfortunately, ordinary right shifting of the data results in the loss of the small counts which are the most important ones. As a result, the measurement is automatically stopped before any overflow occurs. Shorter running times may be completely adequate for many purposes.

As the sampling process proceeds, the count array contains a relative measure of how often each power level occurs. This process treats the output of the a/d converter as a discrete random variable, Y. The count array contains the probability function itself and when properly scaled may be referred to as a probability distribution function (or discrete point probability).

PDF. For the purposes of this discussion the sample point probability for the discrete random variable Y will be called the probability distribution function of Y or PDF. The PDF gives the percentage of time that the power is equal to specific value, y. The percentage ranges from 0 to 100%, and the power extends over the entire dynamic range of the system.
Y is a discrete random variable with a range equal to all possible sampled values of carrier
power. y is a specific power value contained in Y.
PDF expressed as a percentage is:
PDF = P(y) = 100*P[Y=y] where y ranges over all values in Y, $0 \le P(y) \le 100\%$
As samples are continuously taken, the sample space is rescaled to 100%. This conforms to the requirement that all P(y) add up to 100%.

Σ P(y) = 100% where y ranges over all values in Y
The PDF is useful for analyzing the nature of modulating signals. Sustained power levels such as the flat tops of pulses or steps show up as lines. Random noise produces a gaussian shaped curve.

CDF. Another and more useful function which can be derived from the sample count array is the integrated probability density or cumulative distribution function or CDF. For the discreet random variable Y, the CDF is the probability that the power is less than or equal to a specific value, y. The CDF is non-decreasing in y, that is, the graph of CDF versus y cannot have negative slope. The maximum power sample taken will lie at 100%.
CDF expressed as a percentage is:
CDF = Q(y) = 100*P[$Y \le y$] where y ranges over all values in Y, $0 \le Q(y) \le 100\%$

$Q(y_{max}) = 100\%$, and also, just as for PDF above, Σ P(y) = 100%

CCDF. It is often convenient to use the complementary CDF, or CCDF, or 1-CDF, sometimes called the "upper tail area". The CCDF is the probability that the power is greater than a specific power value. CCDF is non-increasing in y and the maximum power sample lies at 0%.
CCDF expressed as a percentage is:
CCDF = 1-Q(y) = 100*P[Y>y] where y ranges over all values in Y
$0 \le 1-Q(y) \le 100\%$, $1-Q(y_{max}) = 0\%$
In a non-statistical peak power measurement the peak-to-average ratio is the parameter which describes the headroom required in linear amplifiers to prevent clipping or compressing the modulated carrier. The meaning of this ratio is easy to visualize in the case of simple modulation in which there is close correspondence between the modulating waveform and the carrier envelope. When this correspondence is not present, the peak-to-average ratio alone does not

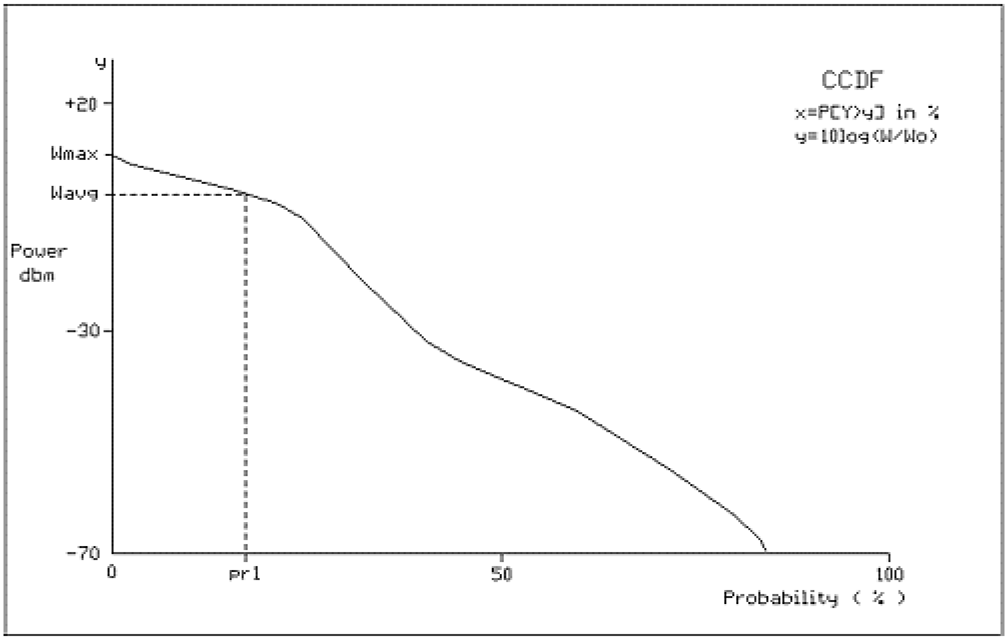

Figure 3. A typical CCDF.

provide adequate information. It is necessary to know what fraction of time the power is above (or below) particular levels. For example, some digital modulation schemes produce narrow and relatively infrequent power peaks which can be compressed with minimal effect. The peak-to-average ratio alone would not reveal anything about the fractional time occurrence of the peaks, but the CDF or CCDF clearly show this information. *See Figs. 3 & 4*

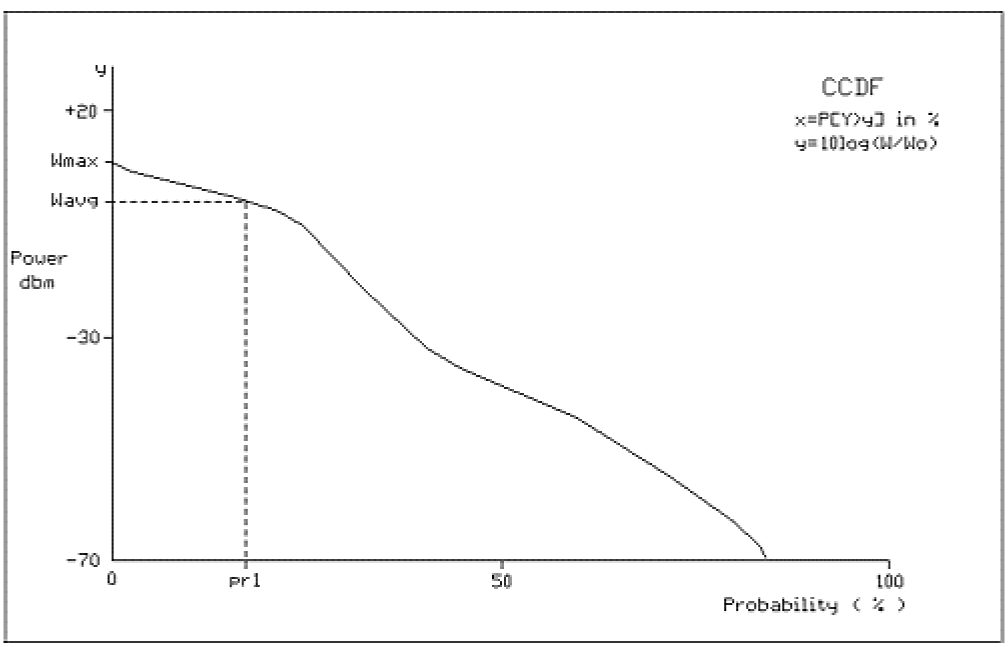

Figure 4. An expanded view of the CCDF in figure 3.

Assume a full length run of one hour plus has been made and the CCDF is analyzed. At CCDF = 0% is the maximum peak power which occurred during the entire run. At CCDF = 1% is the power level which was exceeded only 1% of the time during the entire run. Note that this analysis does not depend upon any particular test signal, nor upon synchronization with the modulating signal and there is no time base involved. In fact, the analysis can be done using actual communication system signals. Normal operation is not disturbed by the need to inject special test signals. This type of analysis is particularly suited to the situation in which the bit error rate (BER) or some other error rate measure is correlated with the percentage of time that the signal is corrupted. If known short intervals of clipping are tolerable, the CCDF can be used to determine optimum transmitter power output. The CCDF is also used to evaluate various modulation schemes to determine the demands that will be made on linear amplifiers and transmitters and the sensitivity to non-linear behavior.

References

Abramowitz and Stegun, Editors, Handbook of Mathematical Functions,NBS Applied Mathematics Series 55, US Dept., of Commerce, 1968.
Bic, J.C. , Duponteil, D. and Imbeaux, J.C., Elements of Digital Communication, John Wiley & Sons, 1991. ISBN 0 471 91571 8
Lee, Y.W., Statistical Theory of Communication, John Wiley & Sons, 1967.